Searching and Sorting

CS10003 PROGRAMMING AND DATA STRUCTURES

Search

Searching

Check if a given element (called key) occurs in the array.

- Example: array of student records; rollno can be the key.

Two methods to be discussed:

- If the array elements are unsorted.
    - Linear search
- If the array elements are sorted.
    - Binary search

Basic Concept of Linear Search

Basic idea:

- Start at the beginning of the array.
- Inspect elements one by one to see if it matches the key.

Time complexity:

- A measure of how long an algorithm runs before terminating.
- If there are *n* elements in the array:
  - Best case:

    match found in first element (1 search operation)
  - Worst case:

    no match found, or match found in the last element (n search operations)
  - Average case: *(n + 1) / 2 search operations*

Linear Search

Function linear_search returns the array index where a match is found.
It returns -1 if there is no match.

```
int  linear_search (int a[], int size, int key)
{
        int pos = 0;
        while ((pos < size) && (a[pos] != key))  pos++;
        if (pos < size)
                return pos;             /* Return the position of match */
        return -1;                      /* No match found */
}
```

Binary Search

Basic Concept

Binary search is applicable if the array is *sorted*.

BASIC IDEA

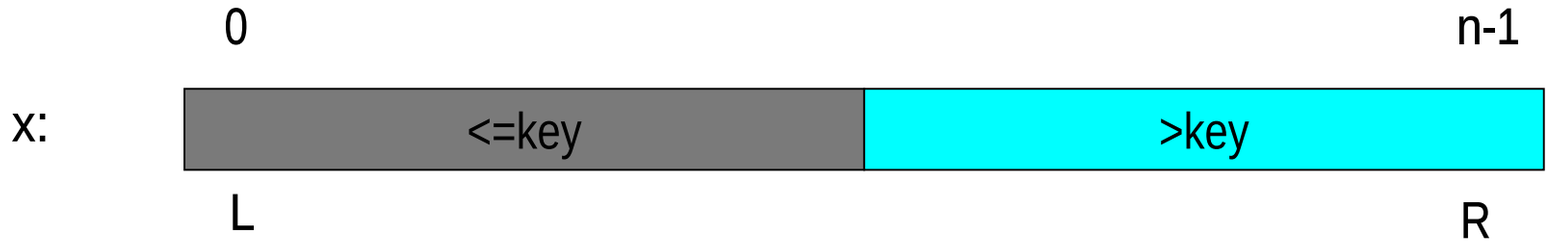
- Look for the target in the middle.
- If you don’t find it, you can ignore half of the array, and repeat the process with the other half.

In every step, we reduce the number of elements to search in by half.

The Basic Strategy

What do we want?

- Find split between values larger and smaller than key:



- Situation while searching:
  - Initially L and R contains the indices of first and last elements.

- Look at the element at index [(L+R)/2].
  - Move L or R to the middle depending on the outcome of test.

Binary Search

```
/* If key appears in x[0..size-1], return its location, pos such that x[pos]==key.
If not found, return -1 */

int bin_search (int x[ ], int size, int key)
{
        int L, R, mid;
        _____ ;
        while ( _____ )
        {
                _____;
        }
        _____ ;
}
```

The basic search iteration

```
int bin_search (int x[ ], int size, int key)
{
        int L, R, mid;
        _____________________;
        while ( _____________ )
        {
                mid = (L + R) / 2;
                if (x[mid] <= key)  L = mid;
                else R = mid;
        }
        _____________________ ;
}
```

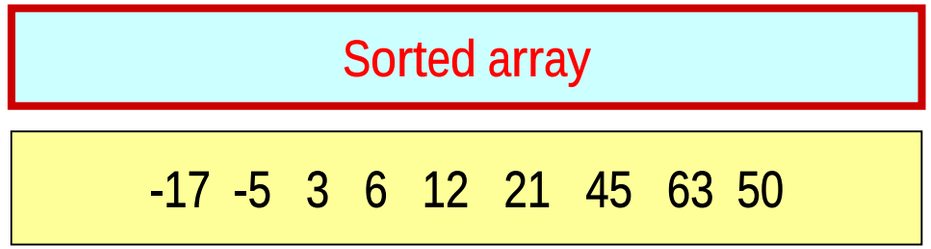
Loop termination criterion

```
int bin_search (int x[ ], int size, int key)
{
        int L, R, mid;
        _____________________;
        while ( L+1 != R )
        {
                mid = (L + R) / 2;
                if (x[mid] <= key) L = mid;
                else R = mid;
        }
        _____________________ ;
}
```

Initialization and Return Value

```
int bin_search (int x[ ], int size, int key)
{
        int L, R, mid;
        L = –1;   R = size;
        while ( L+1 != R )
        {
                mid = (L + R) / 2;
                if (x[mid] <= key) L = mid;
                else R = mid;
        }
        if (L >= 0 && x[L] == key) return L;
        else return –1;
}
```

Binary Search Examples



Trace :

bin_search (x, 9, 3);

bin_search (x, 9, 145);

bin_search (x, 9, 45);

L= –1;	R=9;	x[4]=12;
L= –1;	R=4;	x[1]= –5;
L= 1;	R=4;	x[2]=3;
L=2;	R=4;	x[3]=6;
L=2;	R=3;	return L;

We may modify the algorithm by checking equality with x[mid].

Is it worth the trouble ?

Suppose that the array x has 1000 elements.

Ordinary search

– If *key* is a member of x, it would require 500 comparisons on the average.

Binary search

- after 1st compare, left with 500 elements.
- after 2nd compare, left with 250 elements.
- After at most 10 steps, you are done.

Time Complexity

If there are n elements in the array.

- Number of iterations required: $\log_2 n$

For n = 64 (say).

- Initially, list size = 64.
- After first compare, list size = 32.
- After second compare, list size = 16.
- After third compare, list size = 8.
- …….
- After sixth compare, list size = 1.

$2^k = n$, where k is the number of steps.

$\log_2 64 = 6$

$\log_2 1024 = 10$

Are exactly $\log_2 n$ steps required for all cases?

-17	-5	3	6	12	21	45	63	50

Trace of binsearch(x,9,12):

L= –1; R=9; x[4]=12;
L= 4; R=9; x[6]= 45;
L= 4; R=6; x[5]=21;
L=4; R=5; return L;

We know in first iteration that x[4] = 12. Why not stop then?

Are exactly $\log_2 n$ steps required for all cases?

```
int bin_search (int x[ ], int size, int key)
{
        int L, R, mid;
        L = –1;   R = size;
        while ( L+1 != R )
        {
                mid = (L + R) / 2;
                if (x[mid] <= key) L = mid;
                else R = mid;
        }
        if (L >= 0 && x[L] == key) return L;
        else return –1;
}
```

```
int bin_search_1 (int x[ ], int size, int key)
{
        int L, R, mid;
        L = 0;   R = size-1;
        while ( L <= R )
        {
                mid = (L + R) / 2;
                if (x[mid] == key) return mid;
                if (x[mid] < key) L = mid+1;
                else R = mid-1;
         }
         return -1;
}
```

Sorting

The Basic Problem

Given an array: x[0], x[1], ... , x[size-1] reorder entries so that

x[0] <= x[1] <= . . . <= x[size-1]

- List is in non-decreasing order.

We can also sort a list of elements in non-increasing order.

Example

Original list:

10, 30, 20, 80, 70, 10, 60, 40, 70

Sorted in non-decreasing (ascending) order:

10, 10, 20, 30, 40, 60, 70, 70, 80

Sorted in non-increasing (descending) order:

80, 70, 70, 60, 40, 30, 20, 10, 10

BUBBLESORT

We had studied this earlier. Repeatedly scan the array from left to right and exchange successive elements if they are out of order.

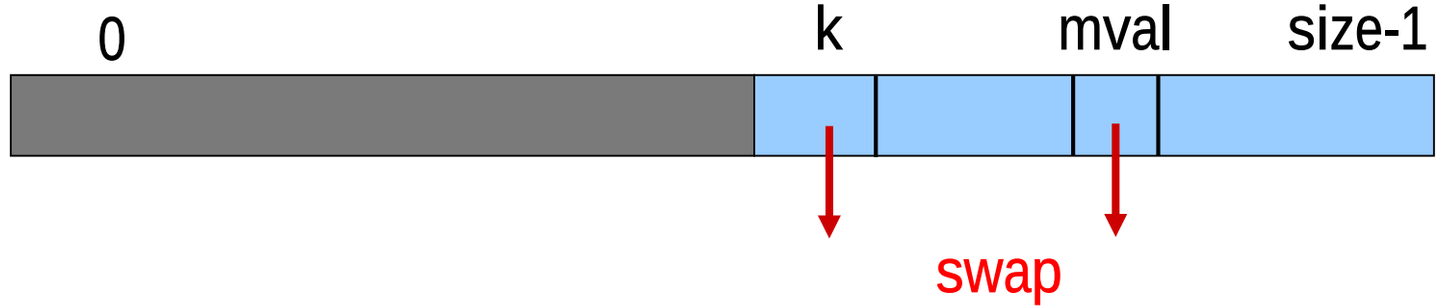
```
void bubbleSort( int arr[ ], int n)
{
        int i, j;

        for (i = 0; i < n – 1; i++)
                for (j = 0; j < n – i – 1; j++)    // Last i elements are already in place
                        if (arr[j] > arr[j + 1]) { temp = arr[j]; arr[j] = arr[j+1]; arr[j+1] = temp; }
}
```

SELECTION SORT

General situation :

Step :

- Find smallest element, mval, in x[k..size-1]
- Swap smallest element with x[k], then increase k.



Subproblem

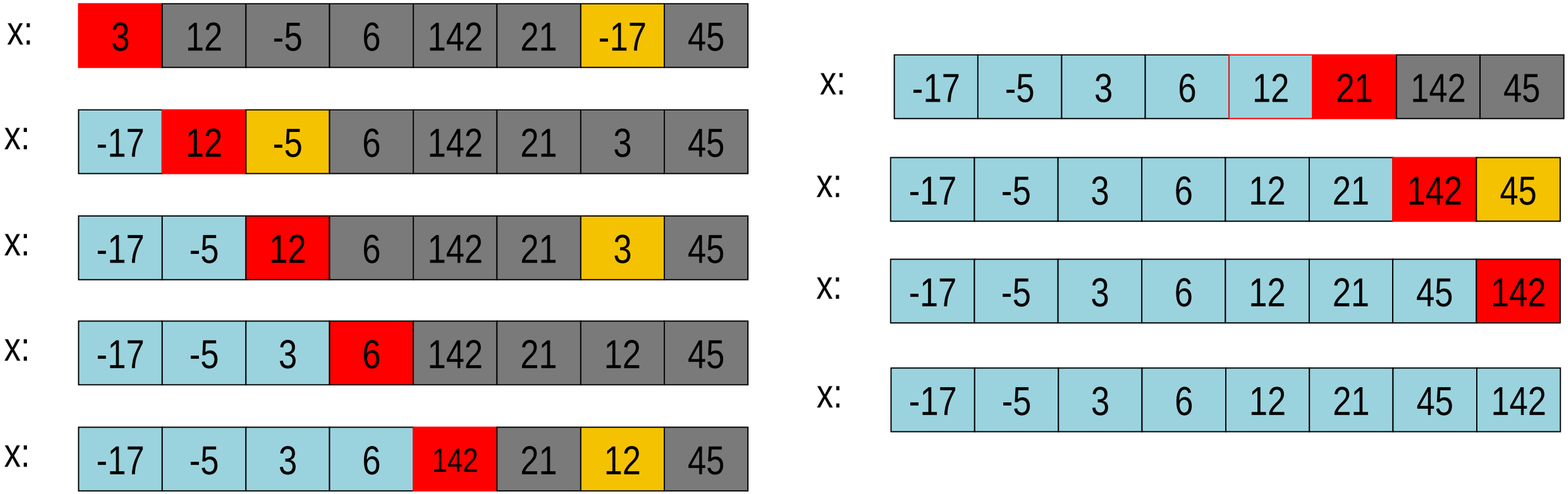
```
/*  Find index of smallest element in x[k..size-1]  */

int min_loc (int x[ ], int k, int size)
{
        int j, pos;

        pos = k;
        for (j=k+1; j<size; j++)
                if ( x[ j ] < x[ pos ] ) pos = j;
        return pos;
}
```

Selection Sort Function

```
/* Sort x[0..size-1] in non-decreasing order */

int sel_sort (int x[ ], int size)
{
        int k, m;

        for (k=0; k<size-1; k++)
        {
                m = min_loc (x, k, size);
                temp = a[ k ]; a[ k ] = a[ m ]; a[ m ] = temp;
         }
}
```

```
/*  Find index of smallest element in x[k..size-1]  */

int min_loc (int x[ ], int k, int size)
{
                int j, pos;

                pos = k;
                for (j=k+1; j<size; j++)
                                if ( x[ j ] < x[ pos ] ) pos = j;
                return pos;
}
```

Example


x: 3 12 -5 6 142 21 -17 45
x: -17 12 -5 6 142 21 3 45
x: -17 -5 12 6 142 21 3 45
x: -17 -5 3 6 142 21 12 45
x: -17 -5 3 6 142 21 12 45
x: -17 -5 3 6 12 21 142 45
x: -17 -5 3 6 12 21 142 45
x: -17 -5 3 6 12 21 45 142
x: -17 -5 3 6 12 21 45 142

Analysis

How many steps are needed to sort *n* items ?

- Total number of steps *proportional* to n^2.
- What is the number of comparisons?

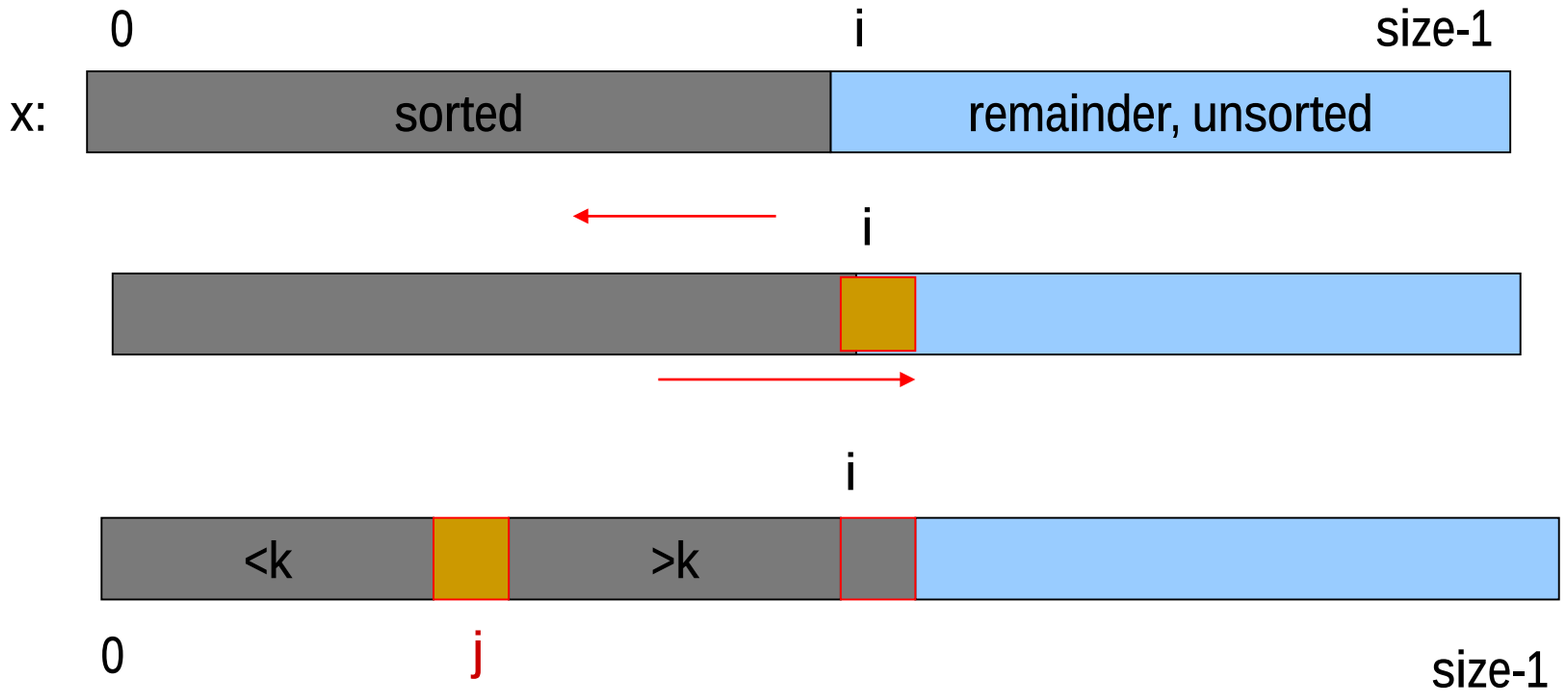
(n-1)+(n-2)+……+1= n(n-1)/2

Of the order of n^2

- Worst Case? Best Case? Average Case?

INSERTION SORT

General situation :



Compare and shift till item = x[i] is larger.

Insertion Sort

```
void insert_sort ( int x[ ], int size )
{
        int i, j, item;

        for (i=1; i<size; i++)
        {
                item = x[i] ;
                for (j=i-1; (j >= 0) && (x[j] > item); j - -)  x[j+1] = x[j];
               x[j+1] = item ;
        }
}
```

Time Complexity

Number of comparisons and shifting:

- Worst case?

  $1 + 2 + 3 + \ldots\ldots + (n-1) = n(n-1)/2$

- Best case?

  $1 + 1 + \ldots\ldots$ to $(n-1)$ terms $= (n-1)$

Some Efficient Sorting Methods

Two of the most popular sorting algorithms are based on divide-and-conquer approach.

- Quick sort
- Merge sort

Basic idea (divide-and-conquer method):

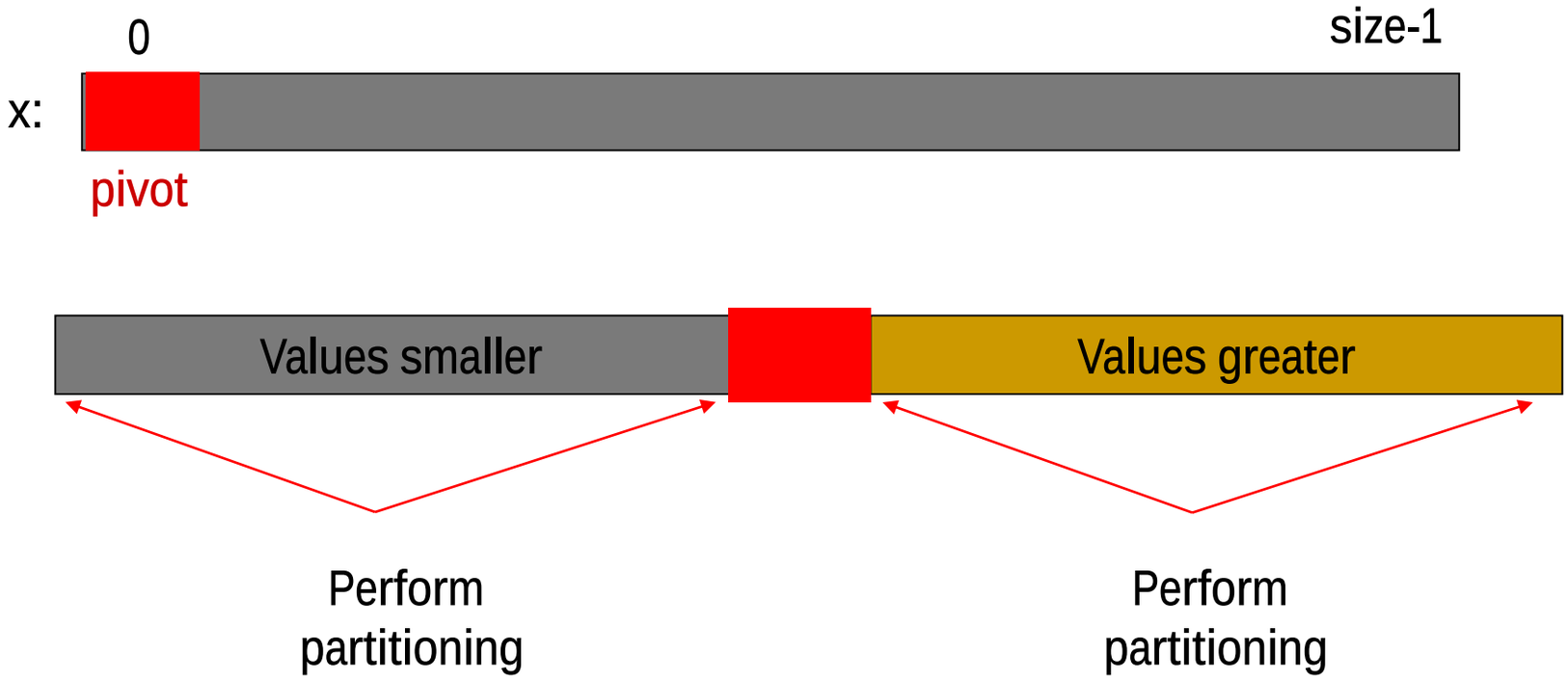
```
sort (list)
{
    if the list has length greater than 1
    {
                    Partition the list into lowlist and highlist;
                    sort (lowlist);
                    sort (highlist);
                    combine (lowlist, highlist);
    }
}
```

QUICKSORT

At every step, we select a *pivot element* in the list (usually the first element).

- We put the pivot element in the *final position* of the sorted list.
- All the elements less than or equal to the pivot element are to the left.
- All the elements greater than the pivot element are to the right.

Partitioning



Preparation

```
void print (int x[ ], int low, int high)
{
        int i;
        for(i=low; i<=high; i++) printf(" %d ", x[i]);
        printf("\n");
}

void swap (int *a, int *b)
{
        int tmp=*a;   *a=*b;    *b=tmp;
}
```

Quicksort

```
void partition ( int x[ ], int low, int high )
{
      int i = low+1,  j = high;
      int pivot = x[low];

      if (low >= high) return;
      while (i<j)  {
            while ((x[i]<=pivot) && (i<high))  i++;
            while ((x[j]>pivot) && (j>low))  j--;
            if (i<j)  swap (&x[i], &x[j]);
       }
       if (j==high)  {
            swap (&x[j], &x[low]);
            partition (x, low, high-1);
       }
      else if (i==low+1)
            partition (x, low+1, high);
      else {
            swap (&x[j], &x[low]);
            partition (x, low, j-1);
            partition (x, j+1, high);
       }
}
```

Time Complexity

Worst case:

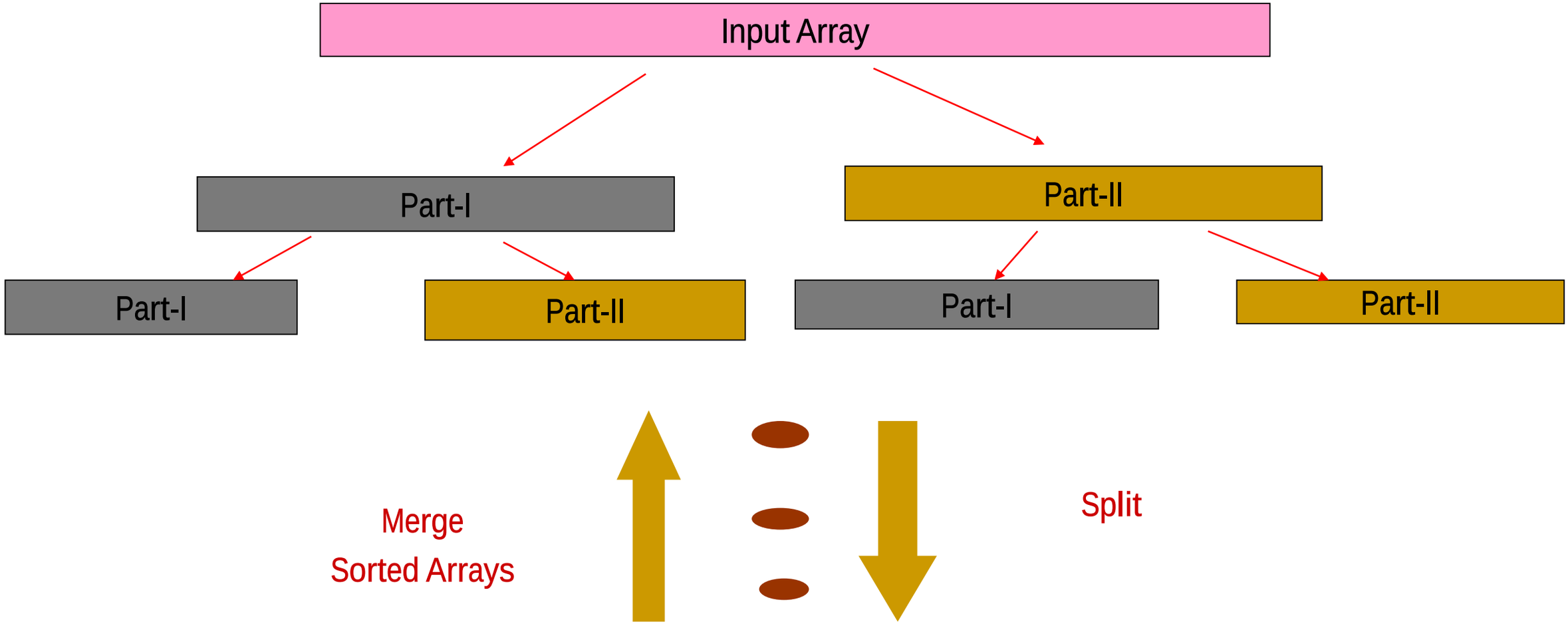
n^2 ==> list is already sorted

Average case:

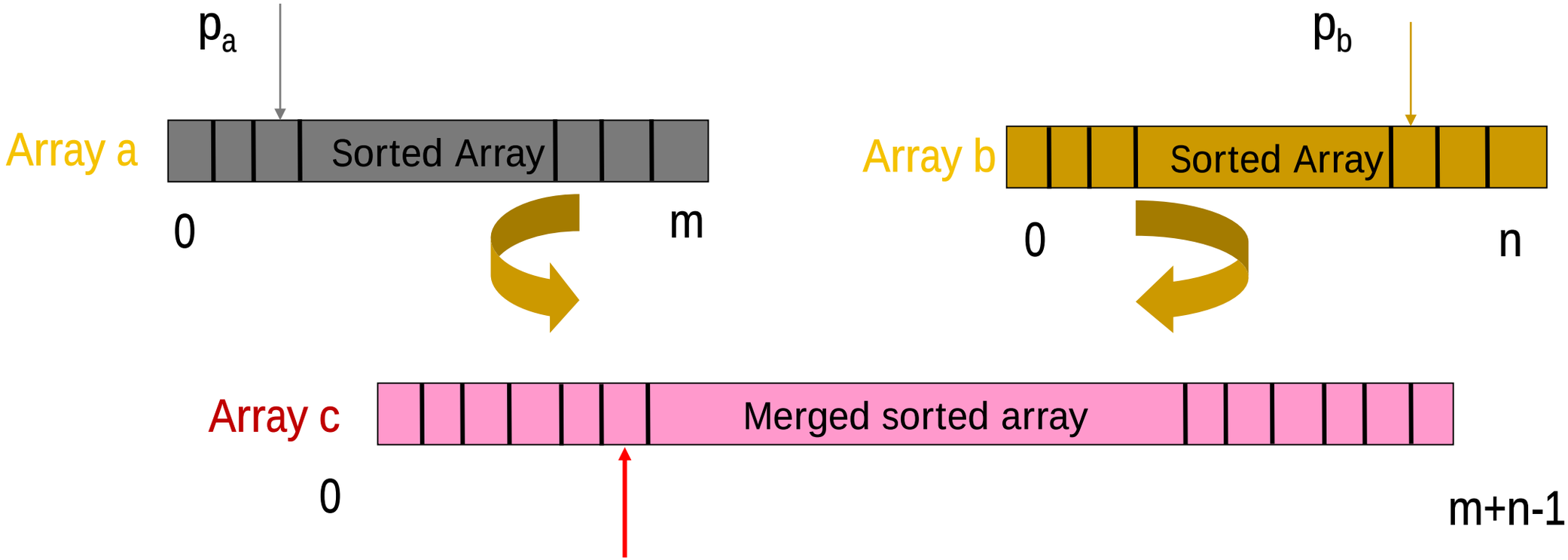
$n \log_2 n$

Statistically, quick sort has been found to be one of the fastest algorithms.

Merge Sort


Input Array
Part-I
Part-II
Part-I
Part-II
Part-I
Part-II
Merge
Sorted Arrays
Split

Merging two sorted arrays



Move and copy elements pointed by p_a if its value is smaller than the element pointed by p_b in (m+n-1) operations; otherwise, copy elements pointed by p_b .

Example

Initial array A contains 14 elements:

- 66, 33, 40, 22, 55, 88, 60, 11, 80, 20, 50, 44, 77, 30

Pass 1 :: Merge each pair of elements

- (33, 66) (22, 40) (55, 88) (11, 60) (20, 80) (44, 50) (30, 70)

Pass 2 :: Merge each pair of pairs

- (22, 33, 40, 66) (11, 55, 60, 88) (20, 44, 50, 80) (30, 77)

Pass 3 :: Merge each pair of sorted quadruplets

- (11, 22, 33, 40, 55, 60, 66, 88) (20, 30, 44, 50, 77, 80)

Pass 4 :: Merge the two sorted subarrays to get the final list

- (11, 20, 22, 30, 33, 40, 44, 50, 55, 60, 66, 77, 80, 88)

```
void merge_sort ( int *a, int n )
{
    int i, j, k, m;
    int *b, *c;

    if (n>1)  {
                k = n/2;   m = n–k;
                b = (int *) calloc(k,sizeof(int));
                c = (int *) calloc(m,sizeof(int));
                for (i=0; i<k; i++)  b[i] = a[i];
                for (j=k; j<n; j++)  c[j– i] = a[j];

                merge_sort (b, k);
                merge_sort (c, m);
                merge (b, c, a, k, m);
                free(b); free(c);
     }
}
```

```
void merge (int *a, int *b, int *c, int m, int n)
{
    int i=0, j=0, k=0, p;

    do  {
                if (a[i] < b[j]) { c[k]=a[i]; i++; }
                else { c[k]=b[j]; j++; }
                k++;
     } while ((i<m) && (j<n));

    if (i == m) {
                for (p=j; p<n; p++)  { c[k]=b[p]; k++; }
    }
    else  {
                for (p=i; p<m; p++)  { c[k]=a[p]; k++; }
    }
}
```

Practice Problems

1. Write a recursive function for binary search.
2. Write iterative version of merge sort.
3. Write merge sort without using additional storage (i.e., extra arrays).
4. You are given a sorted array with entries rotated clockwise by k positions. That is, if the sorted order is a_0, a_1, … , a_{n-1} then the given array to you has the form a_k, a_{k+1}, … , a_{n-1}, a_0, a_1, … , a_{k-1}. Write a variant of binary search on such an array. Assume that k is known.
5. In the previous problem, suppose that k is not given. Write a function that takes the array as input and finds k.